

Searching for the elusive exotic Fulde-Ferrell-Larkin-Ovchinnikov states in Fermi-Fermi mixtures of ultracold quantum gases

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(Dated: June 25, 2014)*

Ultracold two-component Fermi gases with a tunable population imbalance have provided an excellent opportunity for studying the exotic Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) states, which have been of great interest in condensed matter physics. However, the FFLO states have been elusive and not been observed experimentally in Fermi gases in three dimensions (3D), due to their small phase space volume and extremely low temperature required for an equal-mass Fermi gas. Here we explore possible effects of mass imbalance, mainly in a ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture, on the one-plane-wave FFLO phases for a 3D homogeneous case, using a pairing fluctuation theory. We present various phase diagrams related to the FFLO states at finite temperature, throughout the BCS-BEC crossover, and show that a large mass ratio may indeed substantially enhance FFLO type of pairing and make it practical to detect in experiment.

PACS numbers: 03.75.Ss, 03.75.Hh, 67.85.-d, 74.25.Dw

arXiv:1404.5696

The past decade has seen great progress in ultracold atomic Fermi gas studies [1, 2]. With the easy tunability in terms of interaction, dimensionality, population imbalance as well as mass imbalance [1, 2], ultracold Fermi gases have provided a good opportunity to study many exotic quantum phenomena. In particular, the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) states, which were first predicted by Fulde and Ferrell [3] (FF) and Larkin and Ovchinnikov [4] (LO) in an s -wave superconductor in the presence of a Zeeman field almost fifty years ago, have attracted enormous attention in condensed matter physics [5], including heavy-fermion [6], organic [7] and high T_c superconductors [8], nuclear matter [9] and color superconductivity [10], and ultracold Fermi gases [11]. In these exotic states, Cooper pairs condense at a finite momentum \mathbf{q} , with an order parameter of the form of either a plane-wave $\Delta(\mathbf{r}) = \Delta_0 e^{i\mathbf{q}\cdot\mathbf{r}}$ or a standing wave $\Delta(\mathbf{r}) = \Delta_0 \cos(\mathbf{q}\cdot\mathbf{r})$ for the FF and LO states, respectively. Despite many theoretical studies on the FFLO states in equal-mass Fermi gases, both in a 3D homogeneous case [12–16] and in a trap [17–19], the experimental search for these exotic states in atomic Fermi gases still has not been successful [20, 21], largely because they exist only in a *small* region at *very low* temperature in the phase space [12–14, 18]. To find these elusive states, attention has been paid to more complex systems. There have been theoretical investigations in either Fermi-Fermi mixtures [22, 23] or equal-mass Fermi gases with spin-orbit coupling [24, 25] or in an optical lattice [26]. Recently, Stoof and coworkers [22] found an instability toward a supersolid state [27] in a homogeneous ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture in the unitarity and BCS regimes. Using a mean-field theory and the Bogoliubov-de Gennes (BdG) formalism, they have also [23] studied the LO states for the unitary case. However, without including the fluctuation effects, the mean-field theory necessarily overestimates various transition temperatures [1]. In addition, it

is hard to perform stability analysis for various phases in the BdG formalism.

In this paper, we investigate the *one-plane-wave* FFLO states (i.e., the FF states) in a homogeneous ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture, as well as for other mass ratios, as they undergo BCS-BEC crossover. We include the important pairing correlation and pseudogap effects, and find that when the heavy species, ${}^{40}\text{K}$, is the majority, a stable s -wave FFLO phase persists throughout the BCS through BEC regimes, with the population imbalance evolving from small to large. In contrast, when the light species, ${}^6\text{Li}$, is the majority, a stable FFLO phase exists only in the BCS regime. At unitarity, the phase space of stable FFLO states becomes substantially enlarged as the mass ratio increases. The superfluid transition temperature T_c of the FFLO states reaches $0.13T_{F,\uparrow}$ and $0.2T_{F,\uparrow}$ (where $T_{F,\uparrow}$ is the Fermi temperature of the heavy species) for a large mass ratio as in ${}^6\text{Li}$ - ${}^{40}\text{K}$ [28] and ${}^6\text{Li}$ - ${}^{173}\text{Yb}$ [29, 30], respectively. This should be contrasted to $T_c \approx 0.025T_{F,\uparrow}$ for the equal-mass case [12, 24], which is hardly accessible experimentally [21]. Therefore, one may find it realistic to experimentally observe the exotic FFLO states in ultracold Fermi-Fermi mixtures with a large mass ratio.

We consider a three-dimensional (3D) Fermi-Fermi mixture with a short-range contact potential of strength $U < 0$, where momentum \mathbf{k} pairs with $\mathbf{q} - \mathbf{k}$ and thus the Cooper pairs have a nonzero center-of-mass momentum \mathbf{q} . The dispersion of free atoms is given by $\xi_{\mathbf{k},\sigma} = \mathbf{k}^2/2m_\sigma - \mu_\sigma$, where m_σ and μ_σ are the mass and chemical potential for (pseudo)spin $\sigma = \uparrow, \downarrow$, respectively. We set the volume $V = 1$, $\hbar = k_B = 1$. In order to self-consistently treat the pairing fluctuation effects, we use a pairing fluctuation theory previously developed [31] for treating the pseudogap phenomena in high T_c superconductors, which has later been extended successfully to address a variety of ultracold Fermi gas experiments with-

out [1, 32, 33] and with population [34, 35] and/or mass imbalances [36, 37]. It has also been used to study the FFLO states in mass balanced Fermi gases [12, 35]. Here we *apply* this theory to address the FFLO states in the presence of *both* population and mass imbalances in a Fermi-Fermi mixture.

When an FFLO state develops, the rotational symmetry is spontaneously broken with a preferred direction along \mathbf{q} so that the Fermi sphere of spin up and spin down species are no longer concentric, and consequently the pair dispersion minimizes at $\mathbf{q} \neq 0$. Therefore, the Thouless criterion for pairing instability now reads $t_{pg}^{-1}(0, \mathbf{q}) = U^{-1} + \chi(0, \mathbf{q}) = 0$, with $t_{pg}(P)$ being the T -matrix, $\chi(P) = \sum_{K, \sigma} G_{0\sigma}(P - K)G_{\bar{\sigma}}(K)/2$ the pair susceptibility, $G_0(K)$ and $G(K)$ the bare and full Green's functions, respectively, and $G_{0\sigma}^{-1}(K) = i\omega_n - \xi_{\mathbf{k}, \sigma}$. We refer the readers to Ref. [37] for the convention on notations. The self-energy [1] takes approximately the simple BCS-like form, $\Sigma_{\sigma}(K) = -\Delta^2 G_{0\bar{\sigma}}(Q - K)$, with $Q \equiv (0, \mathbf{q})$ and

$$\Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2, \quad \Delta_{pg}^2 \equiv - \sum_P t_{pg}(P), \quad (1)$$

where the superfluid order parameter Δ_{sc} and the pseudogap Δ_{pg} characterize the contributions from the condensate and pairing fluctuations, respectively. Therefore, we have

$$G_{\uparrow}(K) = \frac{u_{\mathbf{k}}^2}{i\omega_n - E_{\mathbf{k}, \uparrow}} + \frac{v_{\mathbf{k}}^2}{i\omega_n + E_{\mathbf{k}, \downarrow}}, \quad (2)$$

$$G_{\downarrow}(K) = \frac{u_{\mathbf{q}-\mathbf{k}}^2}{i\omega_n - E_{\mathbf{q}-\mathbf{k}, \downarrow}} + \frac{v_{\mathbf{q}-\mathbf{k}}^2}{i\omega_n + E_{\mathbf{q}-\mathbf{k}, \uparrow}}, \quad (3)$$

where $u_{\mathbf{k}}^2 = (1 + \xi_{\mathbf{k}\mathbf{q}}/E_{\mathbf{k}\mathbf{q}})/2$, $v_{\mathbf{k}}^2 = (1 - \xi_{\mathbf{k}\mathbf{q}}/E_{\mathbf{k}\mathbf{q}})/2$, $E_{\mathbf{k}\mathbf{q}} = \sqrt{\xi_{\mathbf{k}\mathbf{q}}^2 + \Delta^2}$, and $E_{\mathbf{k}, \uparrow} = E_{\mathbf{k}\mathbf{q}} + \xi_{\mathbf{k}\mathbf{q}}$, $E_{\mathbf{k}, \downarrow} = E_{\mathbf{k}\mathbf{q}} - \xi_{\mathbf{k}\mathbf{q}}$, $\xi_{\mathbf{k}\mathbf{q}} = (\xi_{\mathbf{k}, \uparrow} + \xi_{\mathbf{q}-\mathbf{k}, \downarrow})/2$, $\xi_{\mathbf{k}\mathbf{q}} = (\xi_{\mathbf{k}, \uparrow} - \xi_{\mathbf{q}-\mathbf{k}, \downarrow})/2$. Note that due to the mass imbalance, here $u_{\mathbf{k}}^2 \neq u_{\mathbf{q}-\mathbf{k}}^2$, unlike the equal-mass case. From the number constraint $n_{\sigma} = \sum_K G_{\sigma}(K)$, we can get the number density $n = n_{\uparrow} + n_{\downarrow}$ and population imbalance $\delta n \equiv n_{\uparrow} - n_{\downarrow}$,

$$n = \sum_{\mathbf{k}} \left[\left(1 - \frac{\xi_{\mathbf{k}\mathbf{q}}}{E_{\mathbf{k}\mathbf{q}}}\right) + 2\bar{f}(E_{\mathbf{k}\mathbf{q}}) \frac{\xi_{\mathbf{k}\mathbf{q}}}{E_{\mathbf{k}\mathbf{q}}} \right], \quad (4)$$

$$\delta n = \sum_{\mathbf{k}} \left[f(E_{\mathbf{k}, \uparrow}) - f(E_{\mathbf{k}, \downarrow}) \right], \quad (5)$$

where $\bar{f}(x) = [f(x + \xi_{\mathbf{k}\mathbf{q}}) + f(x - \xi_{\mathbf{k}\mathbf{q}})]/2$ and $f(x)$ is the Fermi distribution function.

Above T_c , the Thouless criterion should be modified by $U^{-1} + \chi(0, \mathbf{q}) = a_0\mu_p$, where μ_p is the *effective* pair chemical potential and a_0 is the coefficient of the linear Ω term in the Taylor expansion of the inverse T -matrix [1] at $\mathbf{p} = \mathbf{q}$, i.e., $t_{pg}^{-1}(\Omega, \mathbf{p}) \approx a_1\Omega^2 + a_0[\Omega - B(\mathbf{p} - \mathbf{q})^2 + \mu_p]$, after analytic continuation. The coefficients a_0 , $B \equiv 1/2M^*$, a_1 and μ_p can be readily derived during the expansion, where M^* is the effective pair mass. This leads to the gap equation

$$\frac{m_r}{2\pi a} = \sum_{\mathbf{k}} \left[\frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1 - 2\bar{f}(E_{\mathbf{k}\mathbf{q}})}{2E_{\mathbf{k}\mathbf{q}}} \right] + a_0\mu_p, \quad (6)$$

where $\mu_p = 0$ at temperature $T \leq T_c$, and $\epsilon_{\mathbf{k}} = k^2/4m_r$ with reduced mass m_r . Note that U has been replaced by the s -wave scattering length a via $U^{-1} = m_r/2\pi a - \sum_{\mathbf{k}} 1/2\epsilon_{\mathbf{k}}$. This also gives the pseudogap equation

$$\Delta_{pg}^2 = \sum_{\mathbf{p}} \frac{b(\tilde{\Omega}_{\mathbf{p}})}{a_0 \sqrt{1 + 4\frac{a_1}{a_0}[B(\mathbf{p} - \mathbf{q})^2 - \mu_p]}}, \quad (7)$$

where $b(x)$ is the Bose distribution function and $\tilde{\Omega}_{\mathbf{p}} = a_0\{\sqrt{1 + 4a_1[B(\mathbf{p} - \mathbf{q})^2 - \mu_p]/a_0} - 1\}/2a_1$ is the pair dispersion. The presence of Δ_{pg} distinguishes Eq. (6) from its mean-field counterpart, despite their formal similarity.

The thermodynamic potential Ω_S consists of fermionic (Ω_F) and bosonic (Ω_B) contributions:

$$\Omega_S = \Omega_F + \Omega_B, \quad (8)$$

$$\Omega_F = -\frac{\Delta^2}{U} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}\mathbf{q}} - E_{\mathbf{k}\mathbf{q}}) - T \sum_{\mathbf{k}, \sigma} \ln(1 + e^{-E_{\mathbf{k}, \sigma}/T}),$$

$$\Omega_B = a_0\mu_p\Delta_{pg}^2 + T \sum_{\mathbf{p}} \ln(1 - e^{-\tilde{\Omega}_{\mathbf{p}}/T}).$$

Momentum \mathbf{q} is determined by minimizing Ω_S at \mathbf{q} , i.e., $\frac{\partial \Omega_S}{\partial \mathbf{q}} = 0$, or equivalently $\frac{\partial \chi(0, \mathbf{p})}{\partial \mathbf{p}}|_{\mathbf{p}=\mathbf{q}} = 0$, which leads to

$$\sum_{\mathbf{k}} \left[\frac{\mathbf{k}}{m_{\uparrow}} (n_{\mathbf{k}\mathbf{q}} + \delta n_{\mathbf{k}\mathbf{q}}) + \frac{\mathbf{q} - \mathbf{k}}{m_{\downarrow}} (n_{\mathbf{k}\mathbf{q}} - \delta n_{\mathbf{k}\mathbf{q}}) \right] = 0, \quad (9)$$

where $n_{\mathbf{k}\mathbf{q}}$ and $\delta n_{\mathbf{k}\mathbf{q}}$ are given by the summands of Eqs. (4) and (5), respectively. Furthermore, the FFLO solutions are subject to the stability condition against phase separation (PS) [12, 38, 39],

$$\frac{\partial^2 \Omega_S}{\partial \Delta^2} \frac{\partial^2 \Omega_S}{\partial \mathbf{q}^2} - \left(\frac{\partial^2 \Omega_S}{\partial \Delta \partial \mathbf{q}} \right)^2 > 0. \quad (10)$$

For the Sarma phase (where $q = 0$), Eq. (10) should be replaced by $\partial^2 \Omega_S / \partial \Delta^2 > 0$.

For the FFLO phases, Eqs. (4)-(7) and (9) along with Eq. (1) are solved for $(\Delta, \mu_{\uparrow}, \mu_{\downarrow}, T_c, \mathbf{q})$ with $\Delta_{sc} = 0$, $(\Delta, \mu_{\uparrow}, \mu_{\downarrow}, \Delta_{sc}, \mathbf{q})$ below T_c , and $(\Delta, \mu_{\uparrow}, \mu_{\downarrow}, \mu_p, \mathbf{q})$ above T_c . For non-FFLO phases, we drop Eq. (9) and set $\mathbf{q} = 0$. We take the heavy (light) species to be spin up (down), and Fermi momentum $k_F = (3\pi^2 n)^{1/3}$. To avoid an artificial jump across population imbalance $p \equiv \delta n/n = 0$ in the phase diagrams, we take $m = (m_{\uparrow} + m_{\downarrow})/2$ and define the Fermi temperature as $T_F = k_F^2/2m$.

Figure 1 shows the calculated T - p phase diagram for a homogeneous ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture at unitarity. Pairing takes place below the pairing temperature T^* (black solid curve). A stable FFLO state exists only when ${}^{40}\text{K}$ is the majority, including FFLO superfluid (yellow shaded area) at low T and FFLO type of pseudogap (PG) phase (gray region) at intermediate T , both at relatively high p . For the latter, there is no phase coherence but the pair dispersion reaches its minimum at momentum $\mathbf{q} \neq 0$. For lower p , FFLO states become unstable

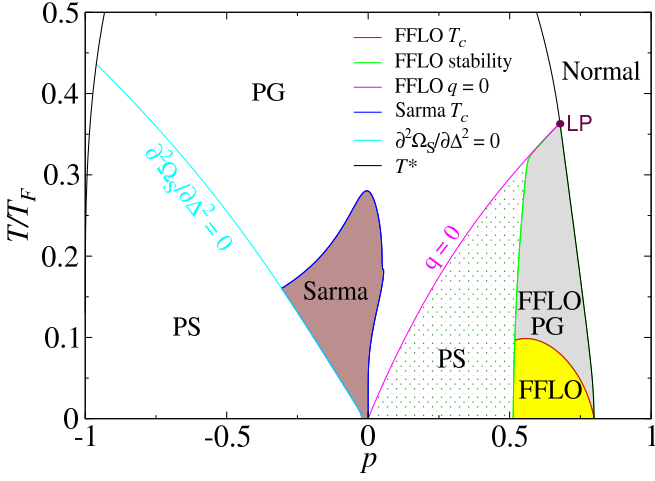


Figure 1. (Color online) T - p phase diagram of a homogeneous ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture at unitarity. Here “PG” and “PS” indicate pseudogapped normal state and phase separation, respectively, and LP labels a possible Lifshitz point. FFLO superfluid (yellow shaded) and pseudogap (gray shaded region) phases exist in the low T and high p regime when ${}^{40}\text{K}$ dominates, while they become unstable in the dotted region. A Sarma superfluid lives in the intermediate T and low p regime (brown shaded region).

and phase separation (PS) take places at low T (dotted region), whereas Sarma superfluid (brown area) and pseudogap states exist at intermediate T . The (green) line that separates the PS and the FFLO phases is given by the stability condition Eq. (10), and the magenta line denotes where \mathbf{q} drops to zero. When ${}^6\text{Li}$ is dominant, i.e., $p < 0$, phase separation dominates the low T region. Note here that, as we focus on the FFLO phases, we do not distinguish superfluid and pseudogap states in the PS regions. The boundary between the Sarma and the PS phases for $p < 0$ is determined by the instability condition $\partial^2\Omega_S/\partial\Delta^2 < 0$. Within the Sarma phase, one finds intermediate temperature superfluid, similar to the equal mass case [34].

Shown in Fig. 2 is the (near)-BCS counterpart of Fig. 1 at $1/k_F a = -1$, with much weaker pseudogap effects. Here we find stable FFLO phases for $p < 0$ as well, when ${}^6\text{Li}$ is the majority. This is different from Refs. [22, 23], which found no LO or supersolid states in the BCS regime for $p < 0$. The $p > 0$ part is rather similar to the unitary case, except that everything moves to lower p and lower T due to weaker pairing strength. For $p < 0$, the $\mathbf{q} = 0$ line splits the PS phase into two regions, representing unstable Sarma (upper) and FFLO (lower part) phases, respectively.

In both Figs. 1 and 2, we have found a Lifshitz-like point (as labeled “LP”), below which “FFLO PG” states emerge. However, since the transition from the unpaired “Normal” phase to the “PG” or “FFLO PG” phase is a crossover rather than a true phase transition, such LPs may not be as pronounced as one would expect for that associated with true phase transitions. This result should be contrasted with that of Refs. [22, 23], which found true LO or supersolid states below the LPs, due

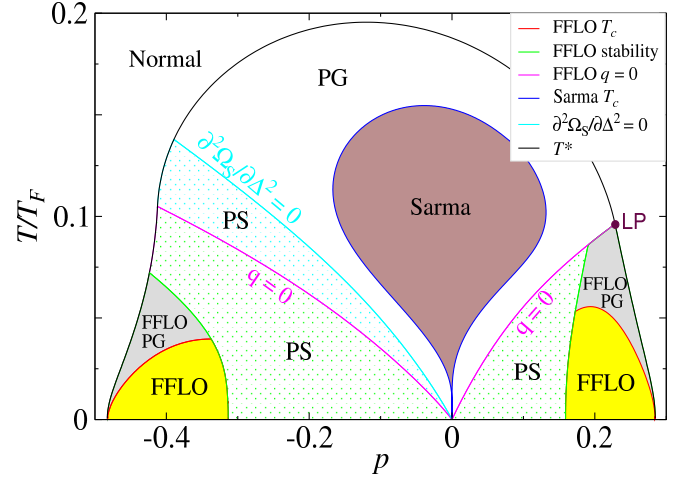


Figure 2. (Color online) T - p phase diagram of a homogeneous ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture at $1/k_F a = -1$, similar to Fig. 1. Here FFLO superfluid (yellow shaded) and pseudogapped (gray shaded regions) states exist for both $p > 0$ and $p < 0$.

to the absence of the pseudogap effects in their treatment.

As the pairing strength grows, the Sarma phase becomes stabilized in a much larger region, especially for $p < 0$ (not shown). However, the stable FFLO states are squeezed towards very low T and very high $p \lesssim 1$, and eventually disappear on the BEC side of the Feshbach resonance. Our result suggests that it is more promising to find FFLO superfluid and pseudogapped phases in the unitary regime.

To ascertain the effect of a varying mass ratio m_\uparrow/m_\downarrow , in Fig. 3 we plot the stable FFLO superfluid phase for $p > 0$ at unitarity for different mass ratios. (The stable FFLO phase for $p < 0$ quickly disappears when $m_\uparrow/m_\downarrow \gtrsim 1.9$). We plot T in units of $T_{F,\uparrow} = k_{F,\uparrow}^2/2m_\uparrow$, the Fermi temperature of the heavy species. The dashed lines for $m_\uparrow/m_\downarrow = 1$ and $40/6$ are the FFLO T_c at and below which the FFLO superfluid is unstable against phase separation. Figure 3 reveals that the phase space of stable FFLO superfluid grows larger and $T_c/T_{F,\uparrow}$ becomes higher as the mass ratio increases. For $m_\uparrow/m_\downarrow = 1$, the maximum FFLO $T_c \approx 0.025T_{F,\uparrow}$ is very low, hardly accessible experimentally. In contrast, it is enhanced up to $0.13T_{F,\uparrow}$ for ${}^6\text{Li}$ - ${}^{40}\text{K}$ and $0.20T_{F,\uparrow}$ for the ${}^6\text{Li}$ - ${}^{173}\text{Yb}$ mixture [29, 30]. This suggests that it is much easier to find experimentally the exotic FFLO superfluid with a large mass ratio. Although the corresponding enhancement of $T_c/T_{F,\downarrow}$ may be less dramatic, we argue that measuring T_c in units of $T_{F,\uparrow}$, which seems to be a natural choice as in Ref. [40], is justified in that the $T \ll T_{F,\uparrow}$ regime is accessible experimentally for ${}^6\text{Li}$ - ${}^{40}\text{K}$. Furthermore, with the recent report of $T \simeq 0.3T_{F,\uparrow}$ [30], it is hopeful that lower T regime can be accessed for ${}^6\text{Li}$ - ${}^{173}\text{Yb}$ as well in the near future.

Shown in Figs. 4 are the calculated p - $1/k_F a$ phase diagrams of a ${}^6\text{Li}$ - ${}^{40}\text{K}$ mixture at $T = 0$ for (a) $p > 0$ and (b) $p < 0$, respectively. When ${}^{40}\text{K}$ is the majority, Fig. 4(a) shows that a narrow (yellow shaded) region of stable FFLO superflu-

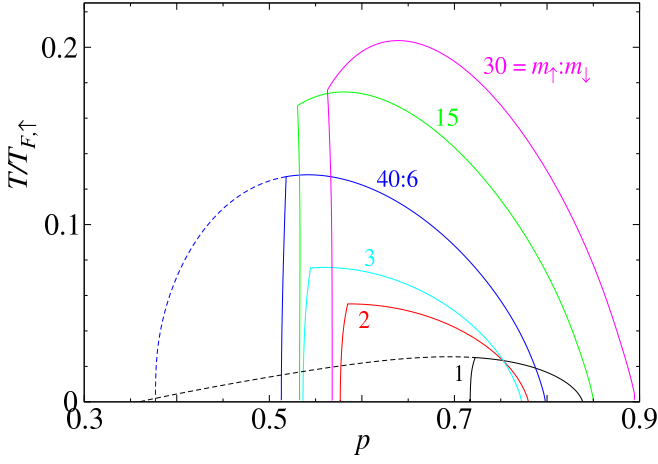


Figure 3. (Color online) T - p phase diagram of stable FFLO superfluid in Fermi-Fermi mixtures with different mass ratios (as labeled) at unitarity. The regions enclosed within solid lines indicate FFLO superfluid that is stable against phase separation, while the dashed lines give the unstable part. For clarity, we only show unstable regions for two representative mass ratios. Note that here for this $p > 0$ regime, we use $T_{F,\uparrow}$ as the energy units.

ids persists from the BCS through the near-BEC regime, up to $1/k_F a \approx 0.55$, as p varies from 0 to 1. Apparently, in the near-BEC regime, the stable FFLO phase exists only at large p . On the other hand, when ${}^6\text{Li}$ is the majority, the stable FFLO phase moves left completely to the BCS side, as shown in Fig. 4(b), in agreement with Figs. 1 and 2. In comparison with the equal-mass case [12], here the stable FFLO region for $p > 0$ is slightly larger, while it becomes smaller for $p < 0$. Here “PS” in both figures labels the regions of FFLO and Sarma superfluids that are unstable against phase separations. In both cases, the FFLO q vector increases from 0 in magnitude as p increases along the stable FFLO phase boundaries.

It is interesting to note that for $p < 0$, due to the left shift of the PS phase, the stable zero T Sarma superfluid phase has extended into the unitary regime ($1/k_F a \gtrsim -0.1$) for small $|p|$, as can also be seen in Fig. 1, where the Sarma phase extends all the way down to $T = 0$ at $p \lesssim 0$. This should be contrasted with the $p > 0$ case and the equal mass case, where zero T Sarma superfluid can be found only when $1/k_F a \gtrsim 1.5$ and $1/k_F a \gtrsim 0.6$ [34], respectively.

While the confining trap in experiment causes inhomogeneity in terms of population imbalances [37, 41, 42], study of the homogeneous case is a necessary first step. (This is also justified in that a homogeneous phase diagram may be obtained experimentally using a tomography technique [40]). In a trap, sandwichlike shell structures will emerge when $p > 0$, with superfluid or pseudogapped normal state in the middle shell [37]. Figure 1 suggests that the FFLO states may be found locally at low T near the shell interfaces where one may find suitable population imbalances.

Finally, we note that while more complex crystalline types

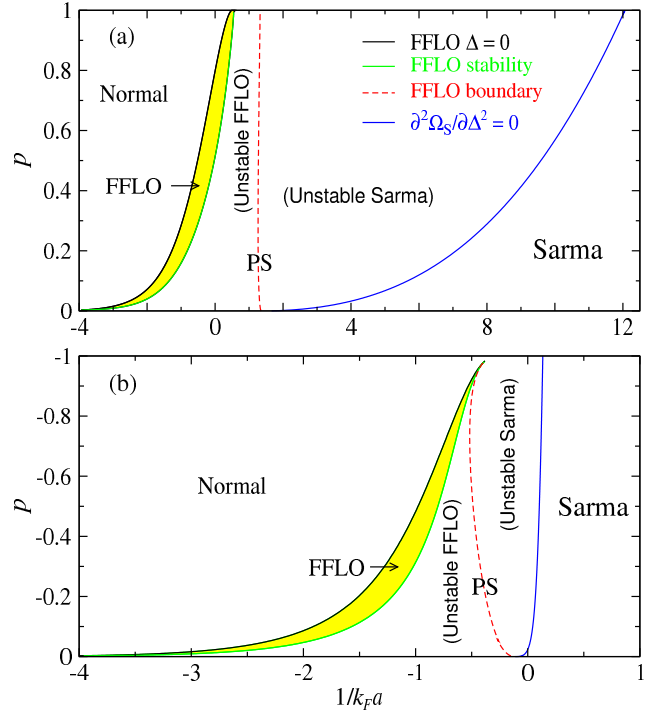


Figure 4. (Color online) Phase diagram of ${}^6\text{Li}$ - ${}^{40}\text{K}$ in the $p - 1/k_F a$ plane at $T = 0$ for (a) $p > 0$ and (b) $p < 0$. Stable FFLO phase lives in the narrow (yellow) shaded regions. Here “PS” labels unstable FFLO and Sarma superfluids, divided by the (red) dashed line.

of FFLO states are expected to have a lower energy and thus may be found within the PS phases in our phase diagrams, a simple theory with proper treatment of pairing fluctuations is yet to be developed. The current theory neglects the *incoherent* background self energy in the pairing channel. We expect this to cause only minor quantitative differences in our results. Further inclusion of particle-hole fluctuations may lead to a shift in the location of unitarity [43].

In summary, our results show that, in order to find the exotic FFLO states in a 3D Fermi gas, it is most promising to explore Fermi-Fermi mixtures with a large mass ratio in the unitary regime, where one expects to see a relatively large phase space volume and a much enhanced superfluid transition temperature when the heavy species is the majority. In addition to the FFLO superfluid states, one may also find FFLO type of pseudogapped normal states. These FFLO states may be detected via collective modes [44], vortices [45], direct imaging [46], rf spectroscopy [47], triplet pair correlations [48], and, most directly, by measuring the pair momentum distribution which should exhibit a peak at a finite q .

This work is supported by NSF of China (Grants No. 10974173 and No. 11274267), the National Basic Research Program of China (Grants No. 2011CB921303 and No. 2012CB927404), NSF of Zhejiang Province of China (Grant No. LZ13A040001).

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